UNIT 1

DIFFERENTIAL CALCULUS

Engineering Mathematics — B.Tech CSE (AI & ML)

CV Raman Global University, Bhubaneswar

Syllabus Coverage
Limit, Continuity and Differentiability of Functions of One Variable
Mean Value Theorems — Rolle's, Lagrange's and Cauchy's
Asymptotes — Cartesian and Polar Forms
Curvature — Cartesian and Polar Forms

Chapter 1: Limit, Continuity and Differentiability

Functions of one variable form the backbone of calculus. Before studying theorems about derivatives, we must firmly understand what it means for a function to be continuous and differentiable.

1.1 Limits

1.1.1 Definition of Limit

Limit: The limit of f(x) as x approaches c is L, written $\lim_{x \to c} f(x) = L$, if f(x) can be made arbitrarily close to L by taking x sufficiently close (but not equal) to c.

1.1.2 One-Sided Limits

- Left-Hand Limit (LHL): $\lim_{x \to c^-} f(x)$ — approach from the left side
- Right-Hand Limit (RHL): $\lim_{x \to c^+} f(x)$ — approach from the right side
- The limit exists if and only if LHL = RHL

1.1.3 Standard Limit Results

$$\lim_{x \to 0} \sin(x)/x = 1$$

$$\lim_{x \to 0} (1 + x)^{(1/x)} = e$$

$$\lim_{x \to a} (x^n - a^n)/(x - a) = n \cdot a^{(n-1)}$$

$$\lim_{x \to 0} (e^x - 1)/x = 1$$

$$\lim_{x \to 0} \log(1 + x)/x = 1$$

Note: A function can have a limit at a point even if it is not defined there. The limit describes the behaviour near the point, not at the point.

1.2 Continuity

1.2.1 Definition of Continuity

A function $f : [a, b] \to \mathbb{R}$ is said to be continuous at a point $c \in (a, b)$ if:

$$\lim_{x \to c} f(x) = f(c)$$

This single condition embeds three requirements: (i) $f(c)$ must exist, (ii) the limit must exist, and (iii) both must be equal.

1.2.2 Continuity on an Interval

- f is continuous on the open interval (a, b) if it is continuous at every point $c \in (a, b)$.
- f is continuous on the closed interval $[a, b]$ if:
  - f is continuous on (a, b)
  - Right continuity at left endpoint: $f(a^+) = f(a)$
  - Left continuity at right endpoint: $f(b^-) = f(b)$

1.2.3 Properties of Continuous Functions

Property	Statement
Boundedness	If f is continuous on $[a, b]$, then f is bounded and attains its bounds at least once.
Intermediate Value	$f(x)$ attains every value between $f(a)$ and $f(b)$ for $x \in [a, b]$.
Zero Existence	If $f(a) \cdot f(b) < 0$, then $\exists\ c \in (a, b)$ such that $f(c) = 0$.
Algebra of Continuity	Sum, difference, product of continuous functions is continuous. Quotient is continuous where denominator $\neq 0$.

1.2.4 Types of Discontinuities

Type	Description	Example

Removable	Limit exists but f(c) is undefined or differs from limit.	$f(x) = \sin(x)/x$ at $x = 0$
Jump	LHL and RHL exist but are unequal.	$f(x) = |x|/x$ at $x = 0$
Infinite	Function or limit becomes unbounded.	$f(x) = 1/x$ at $x = 0$
Oscillatory	Function oscillates infinitely near the point.	$f(x) = \sin(1/x)$ at $x = 0$

1.3 Differentiability

1.3.1 Definition

Derivative: A function $f : [a, b] \to \mathbb{R}$ is differentiable at $c \in (a, b)$ if the following limit exists and is finite:

$$f'(c) = \lim_{x \to c} [f(x) - f(c)] / (x - c)$$

1.3.2 Left and Right Derivatives

$$Lf'(c) = \lim_{h \to 0^-} [f(c+h) - f(c)] / h \qquad \text{(Left Derivative)}$$

$$Rf'(c) = \lim_{h \to 0^+} [f(c+h) - f(c)] / h \qquad \text{(Right Derivative)}$$

f is differentiable at c if and only if $Lf'(c) = Rf'(c)$, and their common value is $f'(c)$.

1.3.3 Differentiability implies Continuity

Note: If f is differentiable at c, then f is continuous at c. The converse is NOT true — $f(x) = |x|$ is continuous at $x = 0$ but not differentiable there.

1.3.4 Relationship Summary Table

Condition	Implies
Differentiable at c	Continuous at c
Continuous at c	Does NOT imply differentiable at c

Discontinuous at c	NOT differentiable at c
$f'(c) > 0$	f is increasing at c
$f'(c) < 0$	f is decreasing at c

Chapter 2: Mean Value Theorems

Mean Value Theorems are powerful bridges between the values of a function and its derivative. They form the theoretical foundation for Taylor's series, numerical methods, and monotonicity analysis.

2.1 Rolle's Theorem

2.1.1 Statement

If $f : [a, b] \to \mathbb{R}$ satisfies all three conditions below, then there exists at least one point $c \in (a, b)$ such that $f'(c) = 0$.

Condition	Requirement
(i) Continuity	f is continuous on the closed interval [a, b]
(ii) Differentiability	f is differentiable on the open interval (a, b)
(iii) Equal Endpoints	$f(a) = f(b)$

$$\exists\ c \in (a, b) \quad \text{such that} \quad f'(c) = 0$$

2.1.2 Geometrical Interpretation

Rolle's Theorem guarantees that the graph of f has at least one horizontal tangent between a and b. Geometrically: if a curve starts and ends at the same height, it must have a peak or valley somewhere in between.

2.1.3 Important Note on Conditions

> **Note:** The conditions of Rolle's theorem are sufficient, not necessary. Even if one condition fails, the conclusion f'(c) = 0 might still hold. The theorem only guarantees the conclusion when ALL three conditions are met.

2.1.4 Classic Examples

Function	Interval	c Found	Remark
$f(x) = x^2 - 1$	[-1, 1]	c = 0	f(-1) = f(1) = 0; f'(0) = 0 ✓
$f(x) = e^{-x} \sin x$	[0, π]	c = π/4	f(0) = f(π) = 0; f'(π/4) = 0 ✓
f(x) = x(x+3)e^(-x/2)	[-3, 0]	c = -2	f(-3) = f(0) = 0; f'(-2) = 0 ✓
$f(x) = x^2 - 2x$	[0, 2]	c = 1	f(0) = f(2) = 0; f'(1) = 0 ✓
f(x) = ex sin x	[0, π]	c = 3π/4	All conditions satisfied ✓

2.1.5 Counter-Examples (Conditions Not Met)

Function	Failed Condition	Observation
f(x) = sin(1/x) on [-1,1]	Discontinuous at 0	f'(x□) = 0 still has roots — conclusion holds accidentally
f(x) = |sin x| on [0,2π]	Not differentiable at x = π	f'(π/2) = 0 and f'(3π/2) = 0 exist anyway
f(x) = sin x on [0, 3π/4]	f(0) ≠ f(3π/4)	f'(π/2) = 0 exists in interval despite condition (iii) failing

2.2 Lagrange's Mean Value Theorem (LMVT)

2.2.1 Statement

If $f : [a, b] \to \mathbb{R}$ is (i) continuous on [a, b] and (ii) differentiable on (a, b), then there exists $c \in (a, b)$ such that:

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f'(c) = [f(b) - f(a)] / (b - a)
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2.2.2 Alternative Form

Writing b = x + h, so c = x + θh for some θ ∈ (0, 1):

$$f(x + h) - f(x) = h \cdot f'(x + \theta h), \quad 0 < \theta < 1$$

2.2.3 Geometrical Interpretation

LMVT states that the slope of the chord joining (a, f(a)) and (b, f(b)) equals the slope of the tangent at some interior point c. In other words, the tangent at c is parallel to the secant through the endpoints.

2.2.4 Proof Sketch

Define $F(x) = f(x) + Ax$, where A is chosen so that $F(a) = F(b)$. This gives:

$$-A = [f(b) - f(a)] / (b - a)$$

Since F satisfies all conditions of Rolle's Theorem, $\exists\ c \in (a, b)$ with $F'(c) = 0$, which gives $f'(c) + A = 0$, hence $f'(c) = [f(b) - f(a)] / (b - a)$.

2.2.5 Corollaries

Corollary	Statement
Constant Function	If $f'(x) = 0$ for all $x \in (a, b)$, then f is constant on [a, b].
Monotone Increasing	If $f'(x) > 0$ for all $x \in (a, b)$, then f is monotonically increasing on [a, b].
Monotone Decreasing	If $f'(x) < 0$ for all $x \in (a, b)$, then f is monotonically decreasing on [a, b].

2.2.6 Worked Examples

Function	Interval	Value of c
$f(x) = (x-1)(x-2)$	[1, 3]	$c = 2$
$f(x) = \log x$	[1, e]	$c = e - 1$
$f(x) = e^x$	[0, 1]	$c = \log(e - 1)$
$f(x) = x - x^3$	[-2, 1]	$c = -1$ or $c = 1/\sqrt{3}$
$f(x) = 1/x$	[1, 4]	$c = 2$
$f(x) = x(x-1)(x-2)$	[0, 1/2]	$c = (6 - \sqrt{21})/6$

2.2.7 Application: Newton's Approximation Method

If $(a + h)$ is an exact root of $f(x) = 0$, by LMVT:

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h ≈ -f(a) / f'(a)      (Newton's Approximation Formula)
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This gives successive approximations to roots of equations. Each iteration typically improves accuracy significantly.

2.3 Cauchy's Mean Value Theorem (CMVT)

2.3.1 Statement

Let $f, g : [a, b] \to \mathbb{R}$ be (i) continuous on $[a, b]$, (ii) differentiable on (a, b), and (iii) $g'(x) \neq 0$ for all $x \in (a, b)$. Then $\exists\ c \in (a, b)$ such that:

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f'(c) / g'(c) = [f(b) - f(a)] / [g(b) - g(a)]
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2.3.2 Relationship to LMVT

Note: Cauchy's Mean Value Theorem is a generalisation of Lagrange's theorem. Setting $g(x) = x$ in CMVT reduces it exactly to LMVT.

2.3.3 Proof Sketch

Let $\varphi(x) = f(x) + A \cdot g(x)$ where A is chosen so $\varphi(a) = \varphi(b)$. This forces:

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-A = [f(b) - f(a)] / [g(b) - g(a)]
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By Rolle's Theorem applied to φ, $\exists$ c with $\varphi'(c) = 0$, giving $f'(c) + A \cdot g'(c) = 0$, and the result follows.

2.3.4 Worked Examples

f(x)	g(x)	Interval	c Found
x^3	x^2	[1, 2]	$c = 14/9$
log x	1/x	[1, e]	$c = e/(e-1)$
sin x	cos x	[a, b]	$c = (a+b)/2$
$\sqrt{x}$	$1/\sqrt{x}$	[a, b]	$c = \sqrt{ab}$ (Geometric Mean)

2.4 Taylor's Theorem (Generalised Mean Value Theorem)

2.4.1 Statement

If f is a function whose (n-1)th derivative is continuous on [a, a+h] and differentiable on (a, a+h), then:

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then the following is true: f(a+h) = f(a) + h·f'(a
          + ... + (h^(n-1)/(n-1)!) ·f^(n-1) (a)
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2.4.2 Forms of the Remainder R_n

Form Name	Remainder R_n Expression
Schlomilch-Roche	$R_n = [h^n(1-\theta)^{(n-p)}] / [(n-1)!p] \cdot f^{(n)}(a + \theta h), \quad 0 < \theta < 1$
Lagrange's Form (p = n)	$R_n = h^n / n! \cdot f^{(n)}(a + \theta h), \quad 0 < \theta < 1$
Cauchy's Form (p = 1)	$R_n = [h^n(1-\theta)^{(n-1)}] / (n-1)! \cdot f^{(n)}(a + \theta h), \quad 0 < \theta < 1$

2.4.3 Maclaurin's Series (Special case: a = 0)

$$f(x) = f(0) + x \cdot f'(0) + (x^2/2!) \cdot f''(0) + (x^3/3!) \cdot f'''(0) + \ldots$$

Function	Maclaurin Series Expansion
e^x	$1 + x + x^2/2! + x^3/3! + x^4/4! + \ldots$
sin(x)	$x - x^3/3! + x^5/5! - x^7/7! + \ldots$
cos(x)	$1 - x^2/2! + x^4/4! - x^6/6! + \ldots$
log(1+x)	$x - x^2/2 + x^3/3 - x^4/4 + \ldots \quad (-1 < x \le 1)$
$(1+x)^n$	$1 + nx + n(n-1)/2!\, x^2 + \ldots$

2.4.4 Hierarchy of Mean Value Theorems

Theorem	Conditions Required	Conclusion
Rolle's Theorem	Continuous on [a,b], differentiable on (a,b), f(a)=f(b)	$\exists$ c: f'(c) = 0

Lagrange's MVT	Continuous on [a,b], differentiable on (a,b)	$\exists$ c: $f'(c) = [f(b)-f(a)]/(b-a)$
Cauchy's MVT	Both f and g are continuous and differentiable; $g'(x) \neq 0$	$\exists$ c: $f'(c)/g'(c) = [f(b)-f(a)]/[g(b)-g(a)]$
Taylor's Theorem	(n-1)th derivative continuous, differentiable	f(a+h) = Σ terms + R□

Chapter 3: Asymptotes

An asymptote is a line that a curve approaches arbitrarily closely as one or both coordinates tend to infinity, without ever actually reaching or crossing the line.

3.1 Types of Asymptotes

Type	Description	Orientation
Horizontal Asymptote	Parallel to the x-axis, y = constant	$x \to \pm\infty$
Vertical Asymptote	Parallel to the y-axis; x = constant	$y \to \pm\infty$
Oblique Asymptote	Neither horizontal nor vertical; $y = mx + c$	x and y both $\to \infty$

3.2 Rectangular (Cartesian) Asymptotes

3.2.1 Horizontal and Vertical Asymptotes

For a curve given by an implicit equation in x and y:

- To find asymptotes parallel to the x-axis: equate to zero the coefficient of the highest power of x in the equation.
- To find asymptotes parallel to the y-axis: equate to zero the coefficient of the highest power of y in the equation.

3.2.2 Example — Rectangular Asymptotes

Curve: $4x^2 + 9y^2 = x^2y^2$

Rewrite as: $4x^2 + 9y^2 - x^2y^2 = 0$

- Coefficient of x^2 (highest power of x): $(4 - y^2) = 0 \rightarrow y = \pm 2$
- Coefficient of y^2 (highest power of y): $(9 - x^2) = 0 \rightarrow x = \pm 3$

Therefore, the asymptotes are $y = 2$ and $y = -2$ (horizontal) and $x = 3$ and $x = -3$ (vertical).

3.3 Oblique Asymptotes

3.3.1 Method (Step-by-Step)

Suppose the asymptote has the form $y = mx + c$. For a curve of degree n, let $\varphi_n(x,y)$ denote the highest-degree terms, $\varphi_{n-1}(x,y)$ the next, and so on.

Step	Action
Step 1	Put $x = 1$, $y = m$ in $\varphi_n(x,y)$, $\varphi_{n-1}(x,y)$, $\varphi_1(x,y)$, etc. to get $\varphi_n(m)$, $\varphi_{n-1}(m)$, ...
Step 2	Solve $\varphi_n(m) = 0$ to get all values of slope m.
Step 3 (Non-repeated m)	Compute c from: $c\cdot\varphi_n'(m) + \varphi_{n-1}(m) = 0$, provided $\varphi_n'(m) \neq 0$.
Step 4 (Repeat m twice)	Solve: $(c^2/2!)\cdot\varphi_n''(m) + (c/1!)\cdot\varphi_{n-1}'(m) + \varphi_{n-2}(m) = 0$
Step 5 (Repeat m, thrice)	Solve: $(c^3/3!)\cdot\varphi_n'''(m) + (c^2/2!)\cdot\varphi_{n-1}''(m) + (c/1!)\cdot\varphi_{n-2}'(m) + \varphi_{n-3}(m) = 0$
Step 6	Write the asymptote as $y = mx + c$.

Note: If $\varphi_n'(m) = 0$ for a non-repeated root m, then no oblique asymptote corresponds to that value of m.

3.3.2 Summary of Oblique Asymptote Formula

$$\text{Asymptote: } y = mx + c, \quad \text{where } c\cdot\varphi_n'(m) + \varphi_{n-1}(m) = 0 \quad \text{(non-repeated m)}$$

3.4 Worked Examples — Oblique Asymptotes

Example 1

Curve: $x^3 - x^2y - xy^2 + y^3 + 2x^2 - 4y^2 + 2x + x + y + 1 = 0$

m value	Type	c value	Asymptote
$m = -1$	Non-repeated	$c = 1$	$x + y = 1$
$m = 1$	Repeated (twice)	$c = (3\pm\sqrt{5})/2$	$2(y-x) = 3+\sqrt{5}$ and $2(y-x) = 3-\sqrt{5}$

Example 2

Curve: $(x^2-y^2)(x+2y) + 5(x^2+y^2) + x + y = 0$

m value	c value	Asymptote
$m = 1$	$c = 5/3$	$y = x + 5/3 \rightarrow 3y - 3x + 5 = 0$
$m = -1$	$c = -5/3$	$y = -x - 5/3 \rightarrow 3x + 3y + 5 = 0$
$m = -1/2$	$c = -25/6$	$y = -x/2 - 25/6 \rightarrow 3x + 6y + 25 = 0$

3.5 Asymptotes of Polar Curves

3.5.1 Method for r = f(θ)

For a polar curve, asymptotes arise when $r \to \infty$ as $\theta \to \alpha$ (some finite angle). The procedure is:

- Find $\theta = \alpha$ such that $r \to \infty$ (i.e., $1/r \to 0$)
- Compute $p = \lim(\theta\to\alpha)\ r \cdot \sin(\theta - \alpha)$
- The asymptote is perpendicular to the radius vector at angle α, at distance p from the pole.

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Polar asymptote at θ = α:  p = lim(θ→α) [1/f(θ)] / [sin(θ - α)]
(by L'Hopital if needed)
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3.5.2 Key Properties

Curve Type	Condition for Asymptote	Approach
$r = a/\sin(n\theta)$	$\sin(n\theta) \to 0$	Find θ where denominator vanishes

r = f(θ) rational in trig	Denominator = 0	Direct substitution method
r = aθ (spiral)	θ → ∞	No finite asymptote; use limits as θ → ∞

Chapter 4: Curvature and Radius of Curvature

Curvature measures how sharply a curve bends at a given point. A straight line has zero curvature; a small circle has large curvature. It is a fundamental concept in differential geometry and physics.

4.1 Definition of Curvature

Curvature (κ): The rate of change of the tangent direction angle ψ with respect to arc length s.

$$\kappa = |d\psi/ds|$$

Here ψ is the angle the tangent makes with the positive x-axis, and s is the arc length parameter. A smaller circle bends more sharply → larger curvature.

4.2 Radius of Curvature

Radius of Curvature (ρ): The reciprocal of curvature: ρ = 1/κ. It is the radius of the osculating (best-fitting) circle at the point.

$$\rho = 1/\kappa = ds/d\psi$$

4.3 Radius of Curvature — Cartesian Form

4.3.1 Standard Formula

$$\rho = [(1 + y_1^2)^{(3/2)}] / |y_2|$$

where $y_1 = dy/dx$ and $y_2 = d^2y/dx^2$.

4.3.2 Special Cases

Condition	Formula
Tangent parallel to x-axis ($y_1 = 0$)	$\rho = 1/\lvert y_2 \rvert$
Tangent parallel to y-axis ($x_1 = 0$)	$\rho = [(1 + x_1^2)^{(3/2)}] / \lvert x_2 \rvert$ (swap roles of x and y)

4.3.3 Worked Examples — Cartesian

Curve	Point / Condition	Radius of Curvature ρ
$y = 4 \sin x - \sin 2x$	$x = \pi/2$	$\rho = 5\sqrt{5}/3$
$y^2 = 8x$ (parabola)	$\rho = 125/2$ given	Point: $(9, \pm3\sqrt{8})$... verify $\rho = 125/2$
$y = c \cosh(x/c)$ (catenary)	Any point	$\rho = y^2/c$ (elegant result)
$x^2/a^2 - y^2/b^2 = 1$ (hyperbola)	Where it touches axes	At $(a,0)$: $\rho = b^2/a$; At $(0,b)$: $\rho = a^2/b$

4.4 Radius of Curvature — Parametric Form

4.4.1 Formula

$$\rho = [(\dot{x}^2 + \dot{y}^2)^{(3/2)}] / \lvert \dot{x}\ddot{y} - \dot{y}\ddot{x} \rvert$$

where $\dot{x} = dx/dt$, $\dot{y} = dy/dt$, $\ddot{x} = d^2x/dt^2$, $\ddot{y} = d^2y/dt^2$.

4.4.2 Worked Examples — Parametric

Curve	Parametric Equations	Result
Cycloid	$x = a(t - \sin t)$, $y = a(1 - \cos t)$	$\rho = 2a\lvert\sin(t/2)\rvert \cdot$ something $\rightarrow \rho = 2\sqrt{(2ay)}$
Astroid	$x = a \cos^3 t$, $y = a \sin^3 t$	$\rho = 3\lvert a \sin t \cos t\rvert = (3a/2)\lvert\sin 2t\rvert$
Ellipse	$x = 4 \cos t$, $y = 3 \sin t$	Min $\rho = 9/4$ at $(\pm4,0)$; Max $\rho = 16/3$ at $(0,\pm3)$
Curve	$x = a(\cos t + t \sin t)$, $y = a \sin t$	$\rho = at$

4.5 Radius of Curvature — Polar Form

4.5.1 Formula

$$\rho = [(r^2 + r_1^2)^{(3/2)}] \;/\; |r^2 + 2r_1^2 - r\cdot r_2|$$

where $r_1 = dr/d\theta$ and $r_2 = d^2r/d\theta^2$.

4.5.2 Worked Examples — Polar

Curve	Result / Key Observation
$r = a(1 + \cos\theta)$ (Cardioid)	$\rho^2/3$ = constant $\rightarrow$ ρ^2 is proportional to r (elegant property)
$r^m = a^m \cos(m\theta)$	$\rho = a^m / (m \cdot r^{m-1}) = r^{(m+1)}/(m\cdot a^m)$ ... simplified to elegant form
$r^2 \cos 2\theta = a^2$ (Lemniscate)	$\rho = a^2/(3r)$ ... purely in terms of r
$r = a\theta$ (Archimedean Spiral)	ρ computed from standard formula

4.5.3 Intersection of Polar Curves — Curvature Ratio

For curves $r = a\theta$ and $r = a/\theta$ intersecting at $\theta = 1$ (where $r = a$):

- Curvature of $r = a\theta$ at intersection: κ_1
- Curvature of $r = a/\theta$ at intersection: κ_2

$$\kappa_1 : \kappa_2 = 3 : 1$$

4.6 Radius of Curvature at Origin — Newton's Method

4.6.1 Applicability

This method applies only when the curve passes through the origin AND has a coordinate axis as tangent at the origin.

Tangent at Origin	Formula
x-axis ($y = 0$ is tangent)	$\rho = \lim_{(y\to 0)} x^2 / (2y)$
y-axis ($x = 0$ is tangent)	$\rho = \lim_{(x\to 0)} y^2 / (2x)$

4.6.2 Examples

Curve	Tangent at Origin	ρ
$y^2 = x^2(a+x)/(a-x)$	x-axis	$\rho = a$
$x^3 + y^3 - 3xy = 0$	y-axis (tangent x=0)	$\rho = 3/2$
$x^3 - y^2 - xy = 0$	y-axis	ρ found by Newton's method

4.7 Summary — Curvature Formulae

Form	Formula for ρ
Cartesian y = f(x)	$[(1 + y_1^2)^{(3/2)}] / \|y_2\|$
Parametric x(t), y(t)	$[(\dot{x}^2 + \dot{y}^2)^{(3/2)}] / \|\dot{x}\ddot{y} - \dot{y}\ddot{x}\|$
Polar r = f(θ)	$[(r^2 + r_1^2)^{(3/2)}] / \|r^2 + 2r_1^2 - r \cdot r_2\|$
At Origin (x-tangent)	$\lim(y \to 0)\ x^2 / (2y)$
At Origin (y-tangent)	$\lim(x \to 0)\ y^2 / (2x)$

Quick Reference — All Key Formulae

Mean Value Theorems — At a Glance

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Rolle's Theorem:     f'(c) = 0,   c ∈ (a,b)   [when f(a) = f(b)]
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Lagrange's MVT:      f'(c) = [f(b) - f(a)] / (b - a)
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Cauchy's MVT:        f'(c)/g'(c) = [f(b)-f(a)] / [g(b)-g(a)]
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Taylor's Theorem: $f(a+h) = \Sigma\ [h^r/r!]\ f^{(r)}(a) + R_n$

Asymptotes — At a Glance

Horizontal/Vertical: Equate highest-power coefficient to zero

Oblique slope: $\varphi_n(m) = 0$ [put $x=1$, $y=m$ in highest degree terms]

Oblique intercept: $c \cdot \varphi_n'(m) + \varphi_{n-1}(m) = 0$ [non-repeated m]

Curvature — At a Glance

Curvature: $\kappa = |d\psi/ds|$ Radius of Curvature: $\rho = 1/\kappa$

Cartesian: $\rho = (1+y_1^2)^{3/2} / |y_2|$

Parametric: $\rho = (\dot{x}^2+\dot{y}^2)^{3/2} / |\dot{x}\ddot{y} - \dot{y}\ddot{x}|$

Polar: $\rho = (r^2+r_1^2)^{3/2} / |r^2+2r_1^2-r \cdot r_2|$

— End of Unit 1: Differential Calculus —